

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

Econometric Estimates of Productivity and Efficiency Change in the Australian Northern Prawn Fishery

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Outline

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

1 Background

2 Production Technology

3 Index Numbers

4 Decomposing TFP indexes

5 Econometric Model

6 The Northern Prawn Fishery

7 Summary

Background

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

- O'Donnell (AJARE 2010; JPA 2012): $\Delta\text{PROF} = \Delta\text{TT}$
 $\times \Delta\text{TFP}$; $\Delta\text{TFP} = \Delta\text{ENV} \times \Delta\text{OTE} \times \Delta\text{OSME} = \dots$; HM
and Fisher TFP indexes; DEA; world agriculture; DPIN
- O'Donnell (AJAE 2012): Lowe TFP index; DEA; US
agriculture
- O'Donnell and Nguyen (2011 CEPA WP): Färe-Primont
TFP index; MLE SFA; hospitals
- O'Donnell (2011 CEPA WP): Färe-Primont TFP index;
Bayesian SFA; US agriculture

Main Tasks

- Estimate the production technology (no prices \Rightarrow distance function; 37 time-series observations \Rightarrow parametric approach)
- Construct a TFP index that satisfies some basic properties (e.g., identity, transitivity, commensurability, proportionality) (distance function \Rightarrow Färe-Primont index)
- Decompose the TFP index into measures of environmental change and efficiency change (Färe-Primont index \Rightarrow $\Delta\text{TFP} = \Delta\text{ENV} \times \Delta\text{OTE} \times \Delta\text{OSE}$)
- Obtain measures of reliability for the components of TFP change (small sample, nonlinear functions \Rightarrow Bayes)

The Production Technology

The production possibilities set:

$$T(z) = \{(x, q) : x \text{ can produce } q \text{ in environment } z\}$$

Assume:

- T1 inactivity is possible
- T2 output set is bounded
- T3 weak essentiality
- T4s strong disposability of outputs
- T5s strong disposability of inputs
- T6 output and input sets are closed

Distance Functions

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

The Shephard output and input distance functions:

$$D_O(x, q, z) = \inf\{\delta > 0 : (x, q/\delta) \in T(z)\}$$

$$D_I(x, q, z) = \sup\{\rho > 0 : (x/\rho, q) \in T(z)\}$$

If T.1, ..., T.6 are satisfied then, among other things,
 $D_O(x, q, z)$ is NN, ND and HD1 in q and $D_I(x, q, z)$ is NN, ND and HD1 in x .

Example

Equivalent representations of a 'regular' technology:

$$T(z) = \{(x, q) : h(q)^r \leq b(z)g(x) \text{ where } 0 < r < \infty\}$$

$$D_O(x, q, z) = h(q)^r / (b(z)g(x))$$

$$D_I(x, q, z) = (b(z)g(x))^{1/r} / h(q)$$

where $h(\cdot)$ is NN, ND and homogeneous of degree $1/r$;
 $g(\cdot)$ is NN, ND and HDr ; and $b(\cdot)$ is positive. This technology
is extended homothetic (EH), extended Hicks neutral (EHN)
and HDr .

Index Numbers

An index that compares q_t with q_s using the latter as the base
(O'Donnell, AJAE 2012):

$$QI_{st} \equiv Q(q_t)/Q(q_s)$$

where $Q(\cdot)$ is NN, ND and HD1. Indexes that have this form satisfy

Q.1 weak monotonicity

Q.2 linear homogeneity

Q.3 identity

:

Q.7 transitivity

TFP Indexes

An index that compares x_t with x_s using the latter as the base:

$$XI_{st} \equiv X(x_t)/X(x_s)$$

where $X(\cdot)$ is NN, ND and HD1. Define $TFP_t = Q(q_t)/X(x_t)$
⇒ the index that compares TFP in periods t and s is

$$TFPI_{st} = \frac{TFP_t}{TFP_s} = \frac{Q(q_t)/X(x_t)}{Q(q_s)/X(x_s)} = \frac{QI_{st}}{XI_{st}}.$$

Again, these indexes satisfy all basic axioms from index theory.
The TFP index is said to be “multiplicatively-complete”.

Färe-Primont Indexes

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

Any function that is NN, ND and HD1 can be used as an aggregator function. If T.1, ..., T.6 are satisfied we can use $Q(q_t) \propto D_O(\mu_x, q_t, \mu_z)$ and $X(x_t) \propto D_I(x_t, \mu_q, \mu_z)$ where μ_x , μ_q and μ_z are arbitrary. Then

$$QI_{st} = \frac{D_O(\mu_x, q_t, \mu_z)}{D_O(\mu_x, q_s, \mu_z)}$$

$$XI_{st} = \frac{D_I(x_t, \mu_q, \mu_z)}{D_I(x_s, \mu_q, \mu_z)}$$

and

$$TFPI_{st} = \frac{D_O(\mu_x, q_t, \mu_z)}{D_O(\mu_x, q_s, \mu_z)} \frac{D_I(x_s, \mu_q, \mu_z)}{D_I(x_t, \mu_q, \mu_z)}$$

Example

If the output and input distance functions are

$$D_O(x_t, q_t, z_t) = h(q_t)^r / (b(z_t)g(x_t))$$

$$D_I(x_t, q_t, z_t) = (b(z_t)g(x_t))^{1/r} / h(q_t)$$

then the Färe-Primont TFP index is

$$TFPI_{st} = \left(\frac{h(q_t)}{h(q_s)} \right)^r \left(\frac{g(x_t)}{g(x_s)} \right)^{1/r}$$

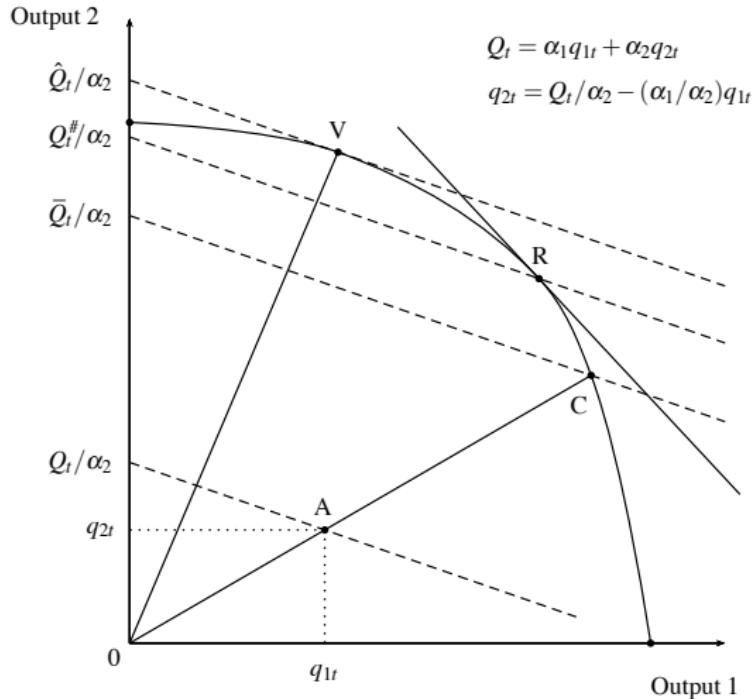
Decomposing TFP Indexes

Any multiplicatively-complete TFP index can be decomposed into any number of measures of environmental change and efficiency change. Output-oriented efficiency measures include:

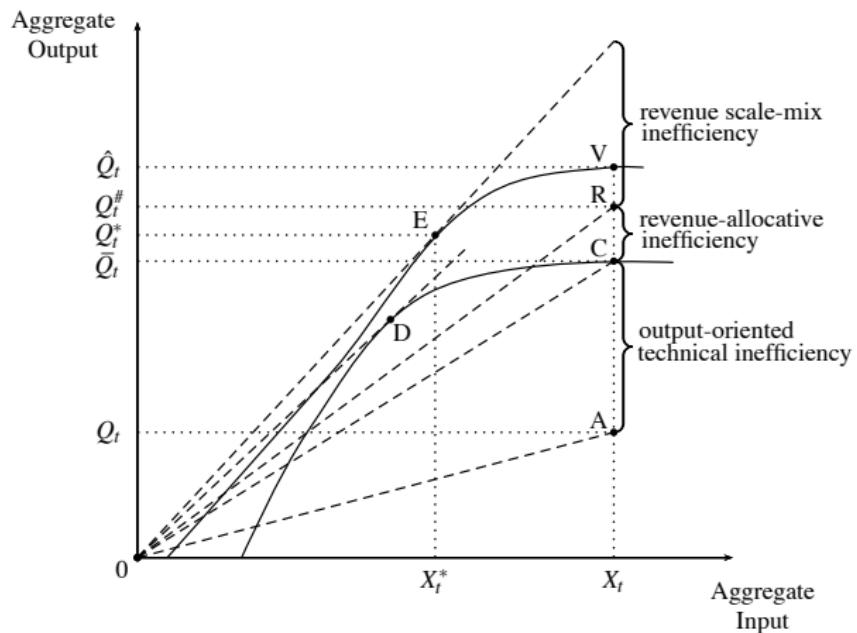
- output-oriented technical efficiency (OTE)
- output-oriented scale efficiency (OSE)
- output-oriented mix efficiency (OME)
- residual output-oriented scale efficiency (ROSE)
- output-oriented scale-mix efficiency (OSME)
- revenue allocative efficiency (RAE)
- ...

Output-Oriented Measures of Efficiency ($N = 2$)

Outline
Background
Production Technology
Index Numbers
Decomposing TFP indexes
Econometric Model
The Northern Prawn Fishery
Summary



Output-Oriented Measures of Efficiency ($N \geq 1$)



Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

Example

If the output and input distance functions are

$$D_O(x_t, q_t, z_t) = h(q_t)^r / (b(z_t)g(x_t))$$

$$D_I(x_t, q_t, z_t) = (b(z_t)g(x_t))^{1/r} / h(q_t)$$

then the Färe-Primont TFP index can be written

$$TFPI_{st} = \left(\frac{b(z_t)}{b(z_s)} \right) \left(\frac{h(q_t)^r}{b(z_t)g(x_t)} \frac{b(z_s)g(x_s)}{h(q_s)^r} \right) \left(\frac{g(x_t)}{g(x_s)} \right)^{(r-1)/r}.$$

Econometric Model

Define

$$v_{dt} \equiv r \ln h(q_t) - \ln b(z_t) - \ln g(x_t) - \ln D_O(x_t, q_t, z_t)$$

$$v_{xt} \equiv \ln g(x_t) - \sum_{m=1}^M \beta_m \ln x_{mt}$$

$$v_{zt} \equiv \ln b(z_t) - \sum_{j=1}^J \gamma_j \ln z_{jt} - \gamma_0$$

$$v_{qt} \equiv \ln \left(\sum_{n=1}^N \alpha_n q_{nt} \right) - r \ln h(q_t)$$

where $\alpha_n \geq 0$, $\sum_n \alpha_n = 1$, $\beta_m \geq 0$ and $\sum_m \beta_m = r$.

Estimating Equation

Without making any assumptions about firm behaviour or the functional form of the output distance function ...

$$\ln Q_t = \gamma_0 + \sum_{j=1}^J \gamma_j \ln z_{jt} + \sum_{m=1}^M \beta_m \ln x_{mt} + v_t - u_t$$

where $Q_t \equiv \sum_n \alpha_n q_{nt}$, $v_t \equiv v_{dt} + v_{zt} + v_{xt} + v_{qt}$ and
 $u_t \equiv -\ln D_O(x_t, q_t, z_t) \geq 0$. The full set of T observations:

$$y = X\beta + v - u$$

where $y = (y_1, \dots, y_T)'$, $y_t \equiv \ln Q_t$ and remaining definitions are obvious. Assume $v \sim N(0, h^{-1} I_T)$.

Estimation

The joint density $p(y|\beta, u, h) = f_N(y|X\beta - u, h^{-1}I_T)$ is not enough to define a sampling density for the $N \times T$ matrix of observed outputs. Follow Fernandez, Koop and Steel (2000, JEmet) and introduce $N - 1$ new random variables (shadow revenue shares):

$$r_{nt}^* = \frac{\partial \ln D_O(x_t, q_t, z_t)}{\partial \ln q_{nt}} = \frac{\alpha_n q_{nt}}{\sum_{k=1}^N \alpha_k q_{kt}}$$

where $p(r_t^*|s) = f_D(r_t^*|s)$. The conditional likelihood function for the matrix of observed outputs:

$$p(Q|\alpha, \beta, h, s, u) = f_N(y|X\beta - u, h^{-1}I_T) \prod_{t=1}^T f_D(r_t^*|s) \prod_{t=1}^T \prod_{n=1}^N \frac{r_{nt}^*}{q_{nt}}$$

Estimation

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

Estimation in a Bayesian framework with a diffuse but proper prior:

$$p(\alpha, \beta, h, s, u) = p(\alpha)p(\beta)p(h)p(s)p(u)$$

where

$$p(\alpha) \sim \text{Dirichlet}$$

$$p(\beta) \sim \text{truncated normal}$$

$$p(h) \sim \text{gamma}$$

$$p(s) \sim \text{gamma}$$

$$p(u|\lambda) \sim \text{gamma}$$

$$p(\lambda) \sim \text{gamma}$$

Estimation

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

Conditional posteriors for use in a Gibbs Sampler:

$p(\beta|\alpha, h, s, \lambda, u, Q) \sim \text{truncated normal}$

$p(h|\alpha, \beta, s, \lambda, u, Q) \sim \text{gamma}$

$p(\lambda|\alpha, \beta, h, s, u, Q) \sim \text{gamma}$

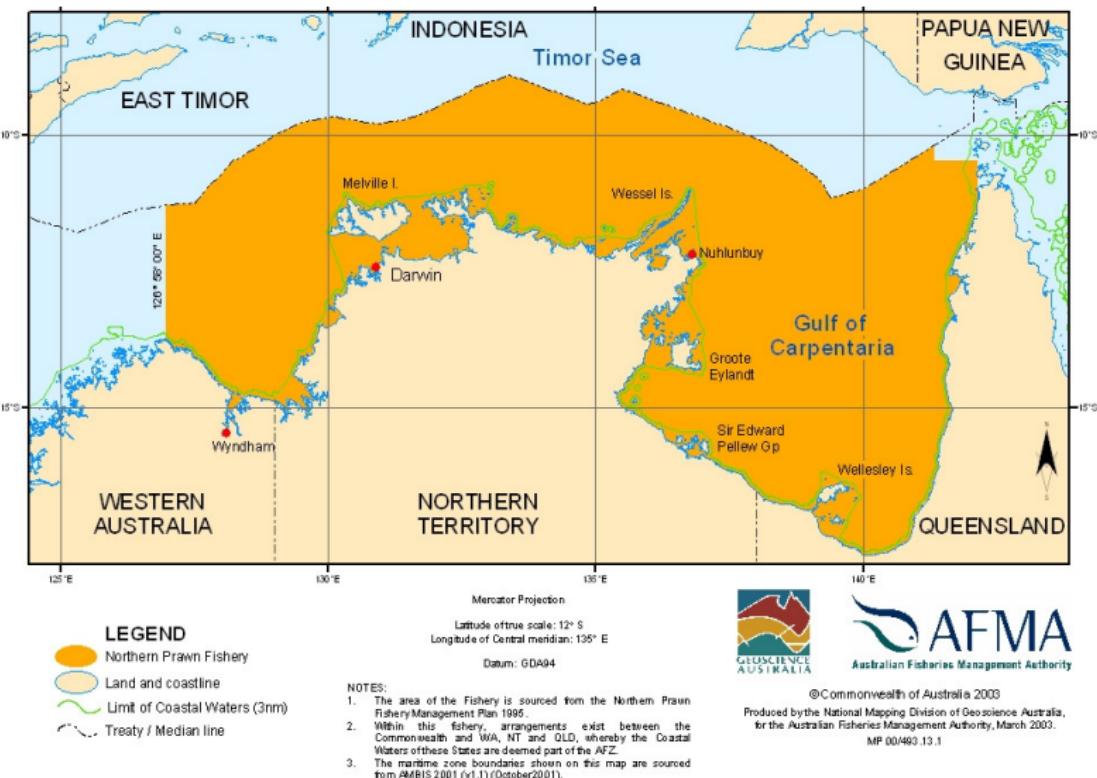
$p(u|\alpha, \beta, h, s, \lambda, Q) \sim \text{truncated normal}$

$p(s_n|\alpha, \beta, h, \lambda, u, Q) \sim \text{messy involving } \Gamma(\sum_n s_n)$

$p(\alpha|\beta, h, s, \lambda, u, Q) \sim \text{messy}$

The Northern Prawn Fishery

Outline
Background
Production Technology
Index Numbers
Decomposing TFP indexes
Econometric Model
The Northern Prawn Fishery
Summary



The Northern Prawn Fishery

Outline

Background

Production
Technology

Index
Numbers

Decomposing
TFP indexes

Econometric
Model

The Northern
Prawn Fishery

Summary

AFMA and CSIRO survey data:

- $T = 37$ years from 1974 to 2010
- $N = 4$ outputs: tonnes of banana, tiger, endeavour, king
- $M = 3$ inputs: vessels, banana effort, tiger effort
- $J = 4$ environment: time, SOI, vessel length, engine size

- 100K burn-in
- 1 million draws, but only kept 1/100 (storage limits)
- constrained $\beta_1, \beta_2, \beta_3, \gamma_3, \gamma_4 \geq 0$ and $\sum_n s_n < 150$

MCMC Chains

Outline

Background

Production
Technology

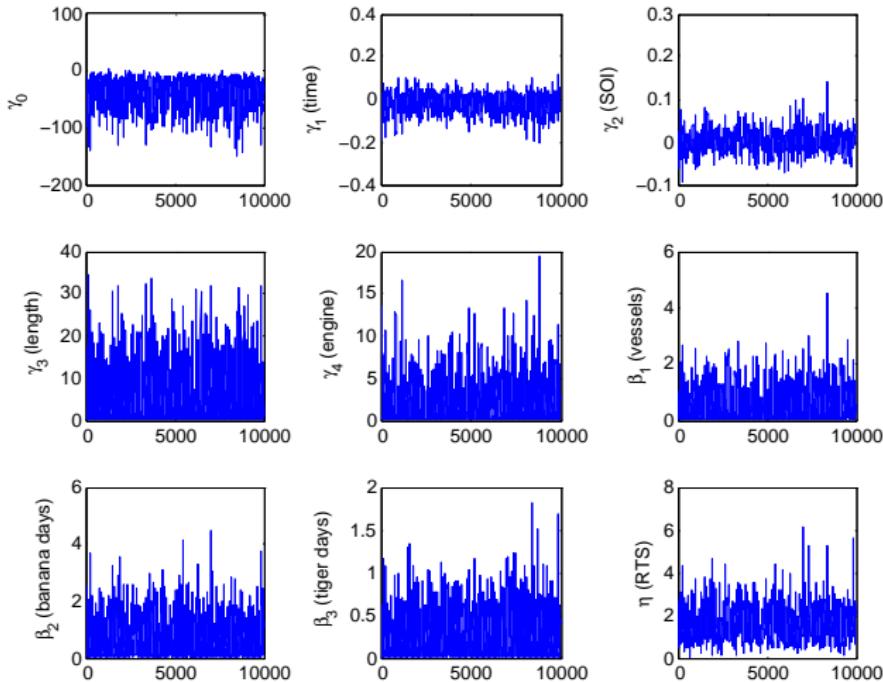
Index
Numbers

Decomposing
TFP indexes

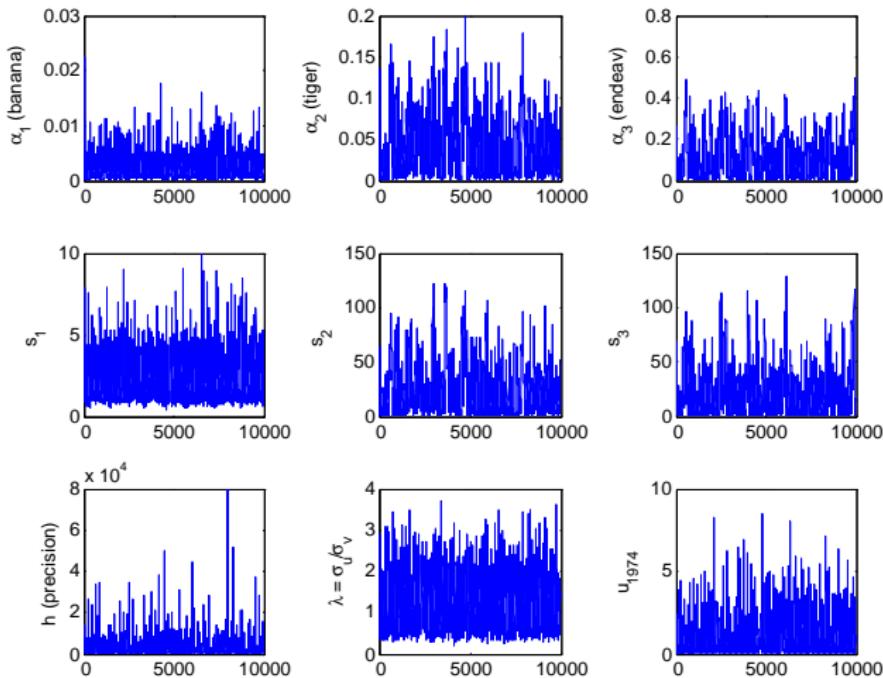
Econometric
Model

The Northern
Prawn Fishery

Summary



MCMC Chains



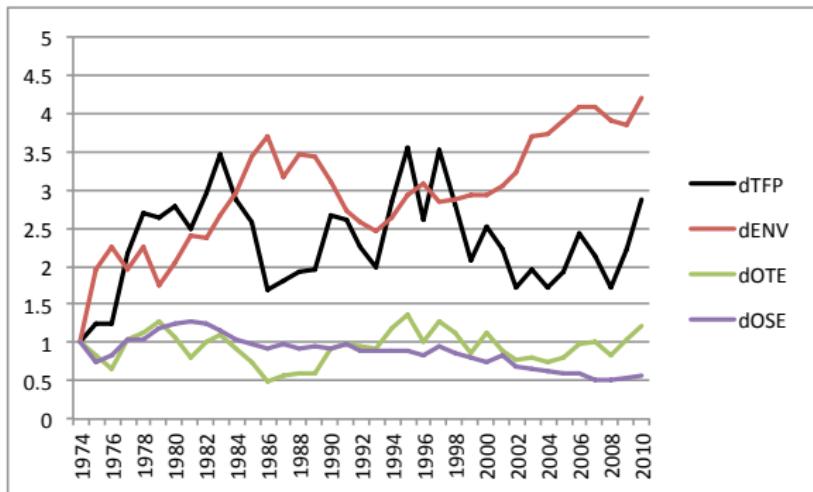
Parameter Estimates

	Mean	St.Dev.	2.5%	97.5%
γ_1	-39.566	20.457	-86.976	-9.089
γ_2 (time)	-0.019	0.035	-0.099	0.040
γ_3 (SOI)	0.002	0.022	-0.039	0.050
γ_4 (length)	6.715	5.230	0.239	19.265
γ_5 (engine)	2.281	2.152	0.069	8.030
β_1 (vessels)	0.571	0.469	0.018	1.714
β_2 (banana effort)	0.789	0.594	0.032	2.158
β_3 (tiger effort)	0.291	0.221	0.011	0.818
$\lambda = \sigma_u / \sigma_v$	1.275	0.491	0.447	2.372
η (RTS)	1.651	0.704	0.533	3.197

Parameter Estimates

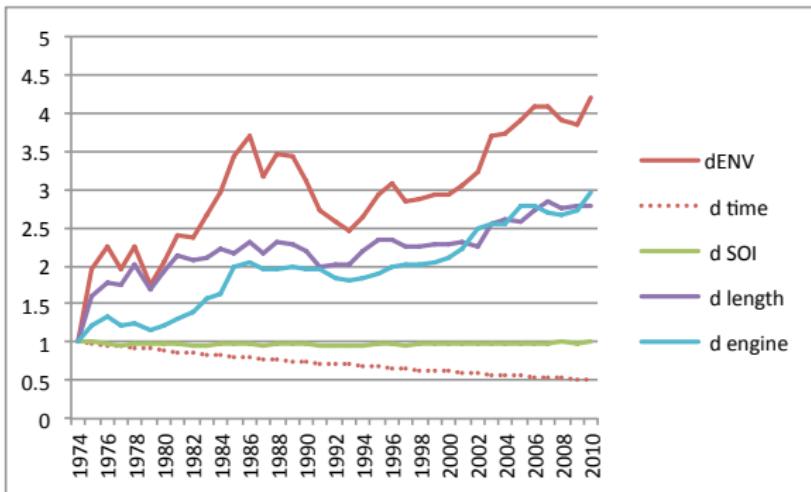
	Mean	St.Dev.	2.5%	97.5%
α_1 (banana)	0.002	0.002	0.000	0.006
α_2 (tiger)	0.042	0.032	0.004	0.124
α_3 (endeavour)	0.120	0.095	0.009	0.352
α_4 (king)	0.836	0.090	0.625	0.965
s_1	2.096	0.869	0.912	4.286
s_2	24.794	20.148	2.318	79.795
s_3	23.129	20.233	1.516	77.063
s_4	8.022	5.653	0.828	22.743
h (precision)	899.620	2640.400	0.232	6850.300

$$\Delta \text{TFP} = \Delta \text{ENV} \times \Delta \text{OTE} \times \Delta \text{OSE}$$



Effects of better environment have been offset by lower scale efficiency

$$\Delta \text{ENV} = \Delta \text{time} \times \Delta \text{SOI} \times \Delta \text{length} \times \Delta \text{engine}$$



Factors causing a deterioration in the production environment over time have been more than offset by larger vessels and increases in engine power

Δ OSE

Outline

Background

Production
Technology

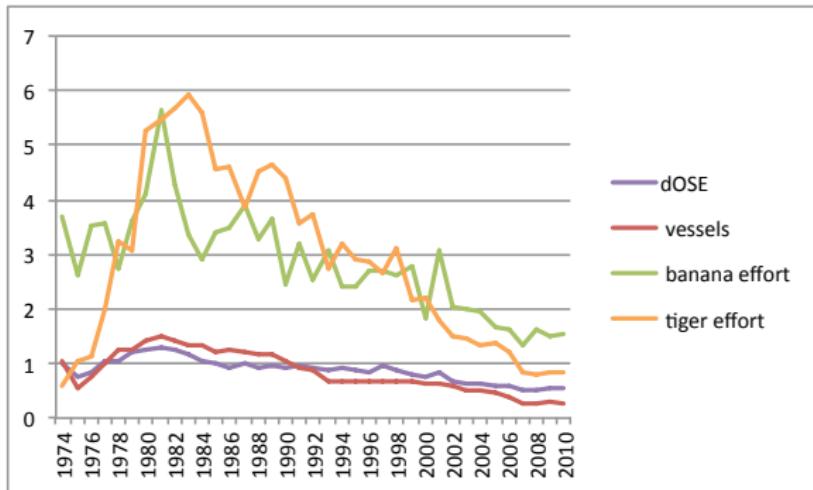
Index
Numbers

Decomposing
TFP indexes

Econometric
Model

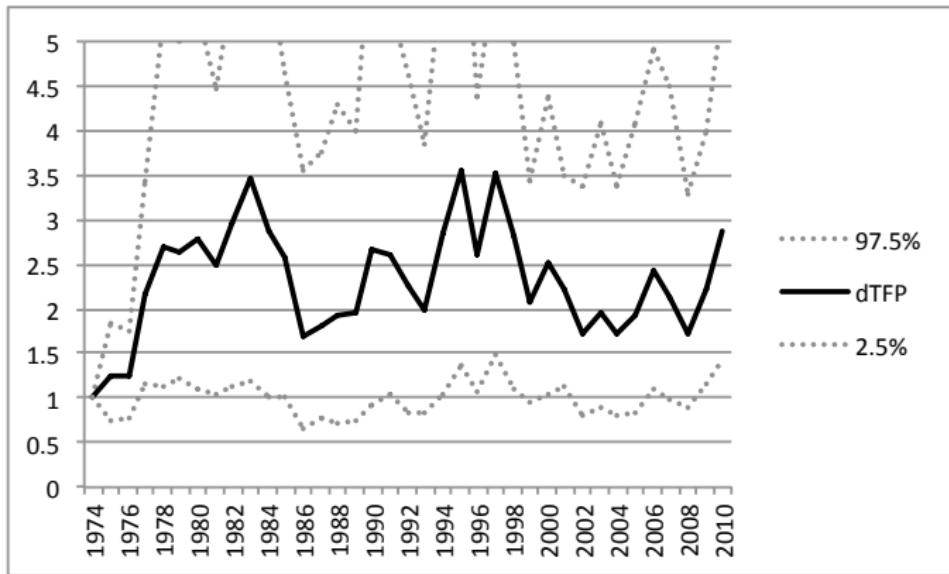
The Northern
Prawn Fishery

Summary



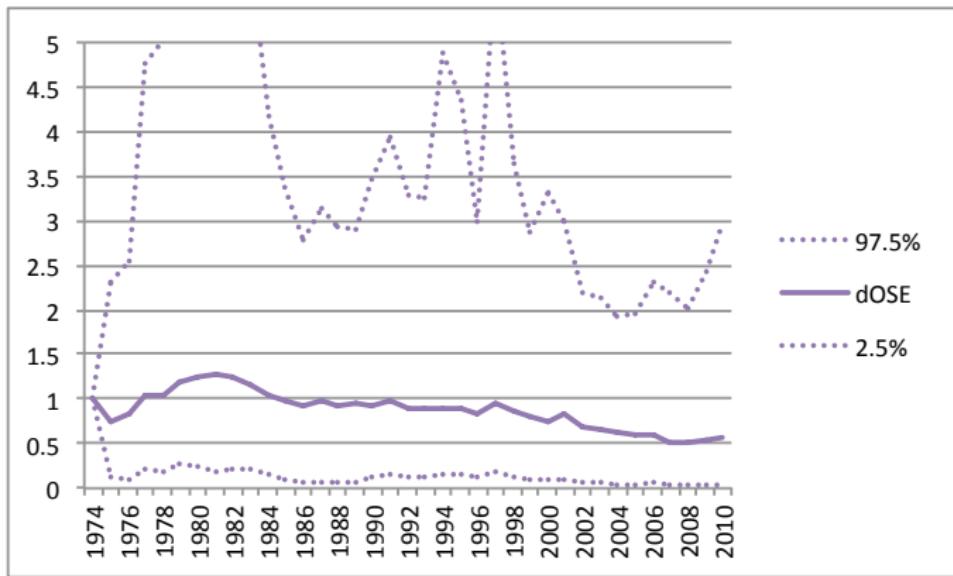
Δ TFP

Outline
Background
Production
Technology
Index
Numbers
Decomposing
TFP indexes
Econometric
Model
The Northern
Prawn Fishery
Summary



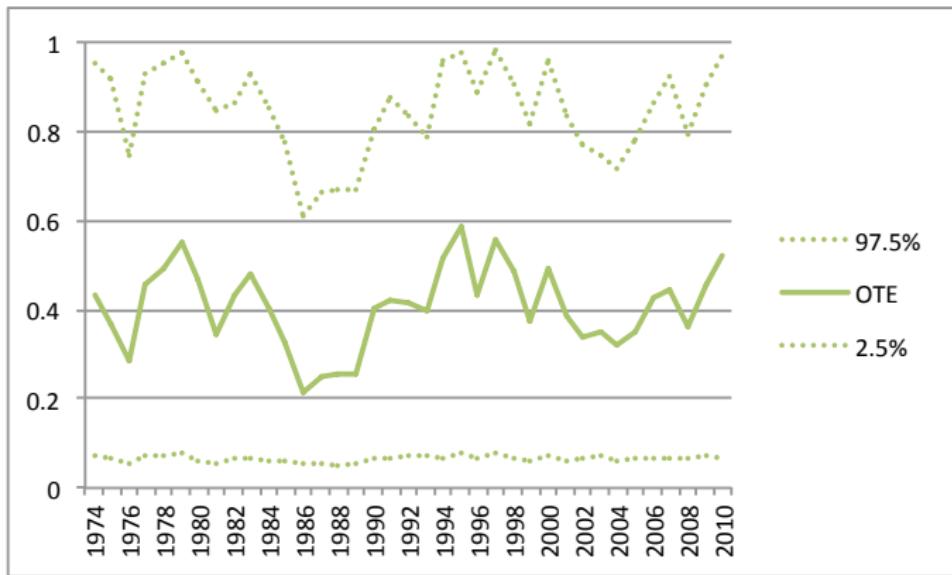
Δ OSE

Outline
Background
Production
Technology
Index
Numbers
Decomposing
TFP indexes
Econometric
Model
The Northern
Prawn Fishery
Summary



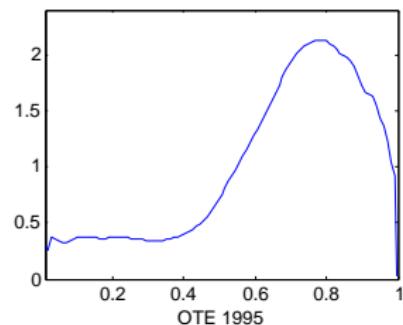
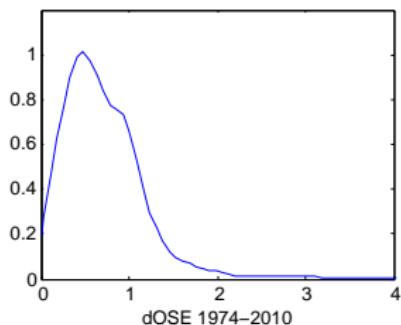
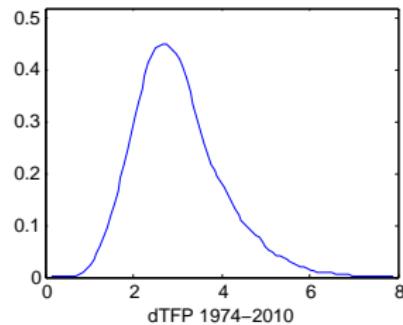
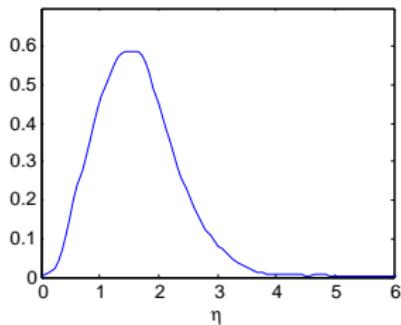
OTE

- Outline
- Background
- Production Technology
- Index Numbers
- Decomposing TFP indexes
- Econometric Model
- The Northern Prawn Fishery
- Summary



Estimated Posterior Pdfs

Outline
Background
Production
Technology
Index
Numbers
Decomposing
TFP indexes
Econometric
Model
The Northern
Prawn Fishery
Summary



Conclusion

- TFP indexes must have certain basic properties (e.g., identity, transitivity, commensurability).
- Indexes for binary comparisons include Laspeyres, Paasche, Fisher, Törnqvist (not Malmquist)
- Indexes for multiple comparisons include Lowe, Geometric Young, Färe-Primont
- $\text{TFP change} = \text{environmental change} \times \text{efficiency change}$ (no residual components, no “effects”)
- We can get reliable estimates of the components of TFP change using small time-series datasets on quantities only
- More information (sample or non-sample) \Rightarrow more efficient/reliable estimates
- More data \Rightarrow DEA (fast and easy using free software)